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A new test for the power transformation model -An analysis of the length of the hospital stay for diabetes patients in Japan -

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Innovation Policy Research Center, The University of Tokyo, Japan http://ipr-ctr.t.u-tokyo.ac.jp/ A new test for the power transformation model -An analysis of the length of the hospital stay for diabetes patients in Japan –

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Abstract: The Box-Cox (1964) transformation model (BC model) is widely used to examine various problems in survival analysis, such as the length of stay (LOS) in a hospital. However, since the error terms cannot have a normal distribution except in the case where the transformation parameter is zero, the likelihood function under the normality assumption is misspecified and the maximum likelihood estimator (BC MLE) cannot be consistent. Nawata (2103) proposed a new consistent estimator of the Box-Cox transformation model. The estimator is a modification of the BC MLE and consistent. Under a certain assumption, the BC MLE can be consistent. In this paper, we first proposed a test whether we can use the BC MLE or not. We then analyze the length of hospital stay of type 2 diabetes patients whose purposes are joining educational programs for managing diabetes at home. The data set of 977 patients collected from 27 general hospitals in Japan is used.

Keywords: Box-Cox transformation model, power transformation model, diabetes, length of stay (LOS)

JEL classification: I18; C25

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1. Introduction

The Box-Cox (1964) transformation model (hereafter, the BC model) is widely used to examine various problems in survival analysis. However, since the error terms cannot have a normal distribution except in the case where the transformation parameter is zero, the likelihood function under the normality assumption (hereafter, the BC likelihood function) is misspecified and the maximum likelihood estimator (hereafter, the BC MLE) cannot be consistent. Alternative methods for the BC model have been proposed by various authors (for details, see Amemiya and Powell (1981), Powell (1996), Yeo and Johnson (2000) and Foster, Tain, and Wei (2001), and Yang (2006)). However, because the simplicity of the model is lost with these versions (Showalter, 1994), these alternatives have not been widely used.

Nawata (2103) proposed a new consistent estimator of the Box-Cox transformation model. Under a certain assumption, the BC MLE can be consistent. Therefore, we first propose a new test whether we can use the BC MLE for the power transformation model (the Box-Cox transformation model excluding the case in which the transformation parameter is zero). Using the newly proposed method, we then analyze the length of stay (LOS) in a hospital for type 2diabetes patients whose purposes are joining educational programs for managing diabetes at home rather than receiving regular medical treatments. Diabetes is now a very important disease in Japan. In 2007, the medical care cost for diabetes was 11.471 billion yen (Ministry of Health, Labour and Welfare, 2009). A large part of medical costs of diabetic patients is determined by the LOS, but the LOS for diabetic patients has not been widely studied. The number of patients in the data set is 977.

2. Model

2.1 A consistent estimator for the power transformation model

We consider the simple power transformation model

$$z_t = x_t'\beta + u_t, \quad z_t = y_t^{\lambda}, \qquad y_t \ge 0, \ t = 1, 2, ..., T,$$
 (1)

where y_t is the LOS, x_t and β are k-th dimensional vectors of explanatory variables and the coefficients, respectively, and λ is the transformation parameter. Random variables $\{u_t\}$ are independent and identically distributed (i.i.d.) and follow a distribution whereby the support is bounded from below, the first and third moments are zero, and the sixth moment exists and is finite (i.e., f(u) = 0 if $u \le -a$ for some a > 0, where f(u) is the probability density function, $E(u_t) = E(u_t^3) = 0$, and $E(u_t^6) = M_6 < \infty$). We do not have to assume a specific distribution, and the model is semiparametric in this sense. $\{x_t\}$ are i.i.d. random variables with the finite third moment. $\{u_t\}$ and $\{x_t\}$ are independently distributed. For the identification of the model, the distribution of x_t and the parameter space of β are restricted so that $\inf(x_t'\beta_0) - a > 0$, where β_0 is the true parameter value of β and $\inf(x_t'\beta) > c$ for some c > 0 in the neighborhood of β_0 . Unlike the case under the normality assumption, $y_t > 0$ under this assumption, and we can obtain a consistent model. (Let $(y_t^{\lambda} - 1)/\lambda = x_t^{*}\beta^* + v_t$ and $v_t = u_t/\lambda$, in which case we obtain the BC model. However, to ensure the asymptotic distribution of the estimator, we only considered the $\lambda \neq 0$ case and did not consider the $\lambda = 0$ case. Therefore, we call this model a power transformation model rather than a BC model.)

Let $\theta' = (\lambda, \beta', \sigma^2)$. The BC likelihood function is given by

$$\log L(\theta) = \sum_{t} \left[\log \phi \{ (z_t - x_t' \beta) / \sigma \} - \log \sigma \right] + \sum_{t} \{ \log \lambda + (\lambda - 1) \log y_t \},$$
(2)

where ϕ is the probability density function of the standard normal assumption and σ^2 is the variance of u_t . The BC MLE is obtained as follows:

$$\frac{\partial \log L}{\partial \lambda} = -\frac{1}{\sigma^2} \sum_{t} (z_t - x_t' \beta) \log(y_t) z_t + \sum_{t} \log(y_t) + n/\lambda = 0, \qquad (3)$$

$$\frac{\partial \log L}{\partial \beta} = \frac{1}{\sigma^2} \sum_{t} x_t (z_t - x_t' \beta) = 0, \text{ and } \frac{\partial \log L}{\partial \sigma^2} = \sum_{t} \frac{(z_t - x_t' \beta)^2 - \sigma^2}{2\sigma^4} = 0.$$

Let $\theta_0' = (\lambda_0, \beta_0', \sigma_0^2)$ be the true parameter value of θ . Since $E[\frac{\partial \log L}{\partial \lambda}|_{\theta_0}] \neq 0$, the BC MLE cannot be consistent. Here, instead of $\partial \log L / \partial \lambda$, we use

$$G_{T}(\theta) = \frac{1}{\sigma^{2}} \sum_{t} (z_{t} - x_{t}'\beta) \{ \log(x_{t}'\beta) + \frac{z_{t} - x_{t}'\beta}{x_{t}'\beta} \} z_{t} + \sum_{t} \{ \log(x_{t}'\beta) + \frac{z_{t} - x_{t}'\beta}{x_{t}'\beta} \} + n = \sum_{t} g_{t}(\theta), \quad (4)$$

 $G_T(\theta)$ is obtained by the approximation of $\partial \log L/\partial \lambda$ as shown in Appendix A. We consider the roots of the equations, as follows:

$$G_T(\theta) = 0, \quad \frac{\partial \log L}{\partial \beta} = 0, \text{ and } \quad \frac{\partial \log L}{\partial \sigma^2} = 0.$$
 (5)

Since $E[G_T(\theta_0)] = 0$, the estimator obtained by Equation (5) is consistent unlike the BC MLE. (For the proof, see Nawata (2013))

Let $\hat{\theta}' = (\hat{\lambda}, \hat{\beta}', \hat{\sigma}^2)$ be the consistent root. The asymptotic distribution of $\hat{\theta}$ is given by $\sqrt{T}(\hat{\theta} - \theta_0) \rightarrow N[0, A^{-1}B(A')^{-1}],$ (6)

where $A = -E[\frac{\partial \ell_{t}(\theta)}{\partial \theta'}|_{\theta_{0}}] = -\begin{bmatrix} A_{11} & A_{12} & A_{13} \\ A_{21} & A_{22} & A_{23} \\ A_{31} & A_{32} & A_{33} \end{bmatrix}$, $A_{11} = E[\frac{\partial g_{t}}{\partial \lambda}|_{\theta_{0}}]$, $A_{12} = E[\frac{\partial g_{t}}{\partial \beta'}|_{\theta_{0}}]$, $A_{13} = E[\frac{\partial g_{t}}{\partial \sigma^{2}}|_{\theta_{0}}]$, $A_{21} = E[\frac{\partial \xi_{t}}{\partial \lambda}|_{\theta_{0}}]$, $A_{22} = E[\frac{\partial \xi_{t}}{\partial \beta'}|_{\theta_{0}}]$, $A_{23} = A_{32}' = E[\frac{\partial \xi_{t}}{\partial \sigma^{2}}|_{\theta_{0}}] = E[\frac{\partial \xi_{t}}{\partial \beta}|_{\theta_{0}}]$, $A_{31} = E[\frac{\partial \xi_{t}}{\partial \lambda}|_{\theta_{0}}]$, $B = E[\ell_{t}(\theta_{0})\ell_{t}(\theta_{0})'] = \begin{bmatrix} B_{11} & B_{12} & B_{13} \\ B_{21} & B_{22} & B_{23} \\ B_{31} & B_{32} & B_{33} \end{bmatrix}$, $B_{11} = E[g_{t}(\theta_{0})^{2}]$, $B_{12}' = B_{21} = E[g_{t}(\theta_{0})\xi_{t}(\theta_{0})]$, $B_{13} = B_{31} = E[g_{t}(\theta_{0})\xi_{t}(\theta_{0})]n$, $B_{22} = E[\xi_{t}(\theta_{0})\xi_{t}(\theta_{0})']$, $B_{23} = B_{32}' = E[\xi_{t}(\theta_{0})\xi_{t}(\theta_{0})]$, $B_{33} = E[\xi_{t}(\theta_{0})^{2}]$, $\ell_{t}(\theta)' = [g_{t}(\theta), \xi_{t}(\theta)', \xi_{t}(\theta)]$, $\xi_{t}(\theta) = \frac{1}{\sigma^{2}}x_{t}(z_{t} - x_{t}'\beta)$, and $\xi_{t}(\theta) = \frac{(z_{t} - x_{t}'\beta) - \sigma^{2}}{2\sigma^{2}}$.

2.2 A test of the "small σ " assumption

The BC MLE is generally inconsistent. However, if $\sigma_0 / x_t / \beta_0 \rightarrow 0$ and $P[y_t < 0] \rightarrow 0$ (in practice, $P[y_t < 0]$ is small enough) under the normality for all observations, the BC MLE performs well and we can use it. Following Bickel and Doksum (1981), we call this the "small σ " assumption. Under the "small σ " assumption the normality assumption is not necessary and we get the "small σ asymptotics" of the BC MLE $\hat{\theta}_{BC} = (\hat{\lambda}_{BC}, \hat{\beta}_{BC}, \hat{\sigma}_{BC}^2)$ given by

$$\sqrt{T}(\hat{\theta}_{BC} - \theta_0) \to N(0, C^{-1}BC^{-1})$$
(11)

where
$$C = -E[\frac{\partial^2 \log L}{\partial \theta \partial \theta'}|_{\theta_0}] = -\begin{bmatrix} C_{11} & C_{12} & C_{13} \\ C_{21} & C_{22} & C_{23} \\ C_{31} & C_{23} & C_{33} \end{bmatrix} C_{11} = E[\frac{\partial^2 \log L}{\partial \lambda^2}|_{\theta_0}], C_{12}' = C_{21} = CE[\frac{\partial^2 \log L}{\partial \lambda \partial \beta}|_{\theta_0}], C_{12}' = C_{21} = CE[\frac{\partial^2 \log L}{\partial \lambda \partial \beta}|_{\theta_0}], C_{12}' = C_{21} = CE[\frac{\partial^2 \log L}{\partial \lambda \partial \beta}|_{\theta_0}], C_{12}' = C_{21} = CE[\frac{\partial^2 \log L}{\partial \lambda \partial \beta}|_{\theta_0}], C_{12}' = C_{21} = CE[\frac{\partial^2 \log L}{\partial \lambda \partial \beta}|_{\theta_0}], C_{12}' = C_{21} = CE[\frac{\partial^2 \log L}{\partial \lambda \partial \beta}|_{\theta_0}], C_{12}' = C_{21} = CE[\frac{\partial^2 \log L}{\partial \lambda \partial \beta}|_{\theta_0}], C_{12}' = C_{21} = CE[\frac{\partial^2 \log L}{\partial \lambda \partial \beta}|_{\theta_0}], C_{12}' = C_{21} = CE[\frac{\partial^2 \log L}{\partial \lambda \partial \beta}|_{\theta_0}], C_{12}' = C_{21} = CE[\frac{\partial^2 \log L}{\partial \lambda \partial \beta}|_{\theta_0}], C_{12}' = C_{21} = CE[\frac{\partial^2 \log L}{\partial \lambda \partial \beta}|_{\theta_0}], C_{12}' = C_{21} = CE[\frac{\partial^2 \log L}{\partial \lambda \partial \beta}|_{\theta_0}], C_{12}' = C_{21} = CE[\frac{\partial^2 \log L}{\partial \lambda \partial \beta}|_{\theta_0}], C_{12}' = C_{21} = CE[\frac{\partial^2 \log L}{\partial \lambda \partial \beta}|_{\theta_0}], C_{12}' = C_{21} = CE[\frac{\partial^2 \log L}{\partial \lambda \partial \beta}|_{\theta_0}], C_{12}' = C_{21} = CE[\frac{\partial^2 \log L}{\partial \lambda \partial \beta}|_{\theta_0}], C_{12}' = C_{21} = CE[\frac{\partial^2 \log L}{\partial \lambda \partial \beta}|_{\theta_0}], C_{12}' = C_{21} = CE[\frac{\partial^2 \log L}{\partial \lambda \partial \beta}|_{\theta_0}], C_{13}' = C_{21} = CE[\frac{\partial^2 \log L}{\partial \lambda \partial \beta}|_{\theta_0}], C_{13}' = CE[\frac{\partial^2 \log L}{\partial \lambda$$

$$C_{13} = C_{31} = E\left[\frac{\partial^2 \log L}{\partial \lambda \partial \sigma^2}|_{\theta_0}\right], C_{22} = E\left[\frac{\partial^2 \log L}{\partial \beta \partial \beta'}|_{\theta_0}\right], C_{23}' = C_{32} = E\left[\frac{\partial^2 \log L}{\partial \beta \partial \sigma^2}|_{\theta_0}\right], \text{ and } C_{33} = E\left[\frac{\partial \log L}{\partial (\sigma^2)^2}|_{\theta_0}\right]$$

Let

$$A^{*} = A^{-1}, \quad A^{*} = \begin{bmatrix} A_{11}^{*} & A_{12}^{*} & A_{13}^{*} \\ A_{21}^{*} & A_{22}^{*} & A_{23}^{*} \\ A_{31}^{*} & A_{32}^{*} & A_{33}^{*} \end{bmatrix}, \quad C^{*} = C^{-1}, \text{ and } C^{*} = \begin{bmatrix} C_{11}^{*} & C_{12}^{*} & C_{13}^{*} \\ C_{21}^{*} & C_{22}^{*} & C_{23}^{*} \\ C_{31}^{*} & C_{32}^{*} & C_{33}^{*} \end{bmatrix}.$$
(12)

 A_{ij}^* and C_{ij}^* are submatrices of A^* and C^* whose locations correspond to A_{ij} and C_{ij} , respectively. Let $\hat{\lambda}_N$ be the proposed estimator of λ . Since $G_T(\theta_0) = \frac{\partial \log L}{\partial \lambda}|_{\theta_0}$ under the "small σ " assumption, we get

$$\sqrt{T}(\hat{\lambda}_{N} - \hat{\lambda}_{BC}) \rightarrow N(0, d),$$
(13)
where $d = p \lim_{n \to \infty} T \cdot V(\hat{\lambda}_{N} - \hat{\lambda}_{BC}) = (A_{11}^{*} - C_{11}^{*})^{2} B_{11} + (A_{12}^{*} - C_{12}^{*}) B_{22} (A_{12}^{*} - C_{12}^{*})' + (A_{13}^{*} - C_{13}^{*})^{2} B_{33}$

+
$$2(A_{11}^* - C_{11}^*)(A_{12}^* - C_{12}^*)B_{12}' + 2(A_{11}^* - C_{11}^*)(A_{13}^* - C_{13}^*)B_{13}.$$

Using $t = \sqrt{T}(\hat{\lambda}_N - \hat{\lambda}_{RC})/\sqrt{\hat{d}}$ as the test statistic, where \hat{d} is the estimator of d, we can test the "small σ " assumption; that is, we can test whether we can successfully use the BC MLE.

3. Analysis of the LOS for the type 2 diabetes patients

3.1 Data

In this section, we analyze the LOS of type 2 diabetic patients whose purposes of hospitalization are joining educational programs for managing diabetes at home rather than receiving regular medical treatments. The data set was collected by the Section of Health Care Economics, Tokyo Medical and Dental University. The survey period was from July 2008 to December 2008. For each patient, dates of hospitalization and discharge from the hospital, date of birth, sex, placement after hospitalization, the International Classification of Diseases-10 (ICD-10) code for the principle disease, purpose of hospitalization, presence of secondary disease and the attending treatment if any, and medical payment amounts were reported. The total number of patients was 3,229 in 67 hospitals and 1,036

patients, 31.4% of all patients, joined the educational program. We use the data set of 977 patients in 27 hospitals (Hp1-27) having 10 or more patients. Generally, it is easier for hospitals to standardize the educational programs than the regular medical treatments. Moreover, most hospitalization can be scheduled in advance for those patients. This means that if the current system works properly, the differences in the LOS are small among hospitals. Thus these cases are considered to be the most suitable candidate for evaluating efficiencies of hospitals. In other words, if the differences in the LOS are large, it may be possible for some hospitals to reduce the LOS through standardization of the educational programs and proper managements of hospitalization schedules for the effective use of medical resources.

For all 27 hospitals, the average length of stay (ALOS) was 14.67 days, the median was 14.0 days, the standard deviation was 6.53 days, the skewness was 1.33, and the kurtosis was 6.44 (the kurtosis is the value where the normal distribution is 0). The maximum ALOS by hospital was 23.3 days (Hp5), and the minimum ALOS was 6.9 days (Hp12). There were very large differences in ALOSs among hospitals. The skewness and kurtosis values were large for some of the hospitals: the large skewness and kurtosis values for certain hospitals imply that some patients remained in these hospitals for a long period of time.

3.2 Results of estimation

We chose the following variables as explanatory variables. The Female Dummy (0: male, 1: female) is used for gender. The proportions of male and female patients were 58.8% and 41.2%, respectively. Since the LOS tends to increase with patient age, we use Age as an explanatory variable. The average age of the patients was 61.0, and its standard deviation was 13.1. Other explanatory variables, representing characteristics of patients, are the Secondary Diseases (numbers of secondary diseases), Complications (numbers of complications), Acute Hospitalization Dummy (acute hospitalization: 1, otherwise: 0), Introduction Dummy (with an introduction of another hospital: 1, otherwise 0), Outpatient Dummy (outpatient of the same hospital before hospitalization: 1, otherwise: 0), and Discharge Dummy (1: discharged to another hospital or facility: 1, otherwise: 0). Among our study subjects, 786 patients had secondary diseases, and the average number per patient was 2.29 for those with secondary diseases. 267 patients had complications, and those patients had 2.05 complications on the average. The numbers of the acute hospitalization patients, outpatients of the same hospital before hospital before hospitalization, and patients discharged to another hospital or facility were 379, 919 and 187, respectively.

For principal disease classifications, dummy variables based on the ICD-10 code E111 are used. For classification, 324 patients had diseases classified under E111, 49 had diseases under E112, 36 had diseases under E113, 75 had diseases under E114, 2 had diseases under E115, 195 had diseases under E116, and 296 had diseases under E117. We used 27 hospital dummies, Hp1,Hp2,...,Hp27 (1: if hospital *i*, 0: otherwise) to represent the influence of hospitals, and a constant term is not included in x_{ii} .

In our model, $x_{ii}'\beta$ of Equation (5) becomes

 $x_{ii}'\beta = \beta_1$ Female Dummy + β_2 Age + β_3 Secondary Diseases + β_4 Complications (8)

+ β_5 Acute Hospitalization Dummy + β_6 Introduction Dummy + β_7 Outpatient Dummy

+ β_8 Discharge Dummy+ $\sum_{\ell} \beta_{\ell} \ell$ -th Principle Disease Dummy+ $\sum_{i} \beta_i$ Hp*i* Dummy

Table 1 presents the results of the estimation by the newly proposed estimator. For the newly proposed estimator, there are two possible problems: i) Equation (5) has multiple solutions, and ii) Equation (5) does not have a solution. However, just one solution exists in this analysis. The estimate of the transformation parameters is $\hat{\lambda}_N = 0.3935$, which is significantly smaller than 1.0; that implies some patients remained in the hospital for a long period of time. We also get $\hat{\lambda}_{BC} = 0.3780$, $\hat{d}/\sqrt{n} = 0.002603$, and $t = \sqrt{T}(\hat{\lambda}_N - \hat{\lambda}_{BC})/\hat{d} = 5.954$. Therefore, the "small σ " assumption is rejected at any reasonable significant level, which means it is not proper to use the BC MLE in this study.

The estimates for the Female Dummy and Age are positive but not significant at the 5% level, so we did not admit the effects of these variables in this study. The estimate of the Secondary Diseases is positive and significant at the 5% level. This means that the secondary diseases make the LOS longer, as expected. The estimate of the Acute Hospitalization Dummy is positive and significant at the 5% level, and the acute hospitalization makes the LOS longer. The estimates of Complications, Introduction Dummy, Outpatient Dummy and Discharge Dummy are not significant at the 5% level, and we could not find any evidence that the LOS depends on these variables. With respect to the principal disease classifications, none of the other estimates is significant at the 5% level. This may related to the fact that the purpose of hospitalization is joining the educational programs and not medical treatments.

For the estimates of the hospital dummies, the maximum and minimum values are 2.892(hp5) and 1.689 (hp12), respectively. The difference between these two values is much larger than the estimates of the other variables. Thus, despite the exclusion of the effects of patient characteristics, surprisingly large differences remain among hospitals. For the effective use of medical resources, it may be necessary for some hospital to revise the current the educational programs and hospitalization schedules to reduce the LOS.

4. Conclusion

The BC model is widely used to examine various problems in survival analysis. However, the BC MLE cannot be consistent. In this paper, we first proposed a test whether we can use the BC MLE or not. Using the newly proposed method, we then analyze the length of stay (LOS) in a hospital for type 2diabetes patients whose purposes are joining educational programs. The number of patients in the data set is 977. The variables found to affect the LOS were the number of secondary diseases and acute hospitalization. We found large differences in the LOS among hospitals, even after eliminating the influence of patient characteristics and principal disease classifications.

The medical information is computerized in many hospitals in Japan. To evaluate and improve the medical payment system in Japan more precisely, it is necessary to analyze data sets by a proper model. It is also necessary to analyze other important cases such as cancer, cardiac infarction, and stroke. These are subjects to be analyzed in future studies.

Appendix A: Approximation of $\partial \log L / \partial \lambda$

Here,

$$\frac{\partial \log L}{\partial \lambda}|_{\theta_0} = -\frac{1}{\sigma_0^2} \sum_{t} \log(y_t) z_t^* u_t + \sum_{t} \log(y_t) + n/\lambda, \tag{9}$$

where $z_t^* = y_t^{\lambda_0}$. If $|u_t / x_t' \beta_0|$ is small, we get

$$\log y_t = \frac{1}{\lambda_0} \log(x_t' \beta_0 + u_t) = \frac{1}{\lambda_0} \{ \log(x_t' \beta_0) + \log(1 + \frac{u_t}{x_t' \beta_0}) \} \approx \frac{1}{\lambda_0} \{ \log(x_t' \beta_0) + \frac{u_t}{x_t' \beta_0} \}.$$
(10)

Therefore, if $\sigma_0 / x_t / \beta_0 \approx 0$ for all observations, we get

$$\frac{\partial \log L}{\partial \lambda}\Big|_{\theta_0} \approx -\frac{1}{\sigma_0^2 \lambda_0} \sum_{t} [x_t' \beta_0 \log(x_t' \beta_0) u_t + \{1 + \log(x_t' \beta_0)\} u_t^2 + \frac{u_t^3}{x_t' \beta_0}]$$

$$+ \frac{1}{\lambda_0} \sum_{t} \{\log(x_t' \beta_0) + \frac{u_t}{x_t' \beta_0}\} + n/\lambda_0 = \frac{1}{\lambda_0} G_T(\theta_0).$$

$$(11)$$

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Variable	Estimate	Standard Error	t-value	Variable	Estimate	Standard Error	t-value
Female Dummy	0.00292	0.06407	0.0455	Hospital Dummies			
Age	0.00202	0.00211	0.9573	hp7	2.3049	0.1134	20.321
Secondary Diseases	0.07677	0.03340	2.2988 *	hp8	2.1572	0.1405	15.354
Complications	0.02897	0.03207	0.9031	hp9	2.4663	0.1287	19.164
Acute Hospitalization Dummy	0.18773	0.08431	2.2268 *	hp10	1.9028	0.1100	17.298
Introduction Dummy	0.05317	0.07544	0.7048	hp11	2.2958	0.0956	24.012
Outpatient Dummy	-0.08759	0.09218	-0.9503	hp12	1.6892	0.2149	7.860
Discharge Dummy	-0.05567	0.09018	-0.6173	hp13	2.5280	0.1435	17.614
Principle Disease Dummies				hp14	2.3730	0.1009	23.527
E112	0.0880	0.1508	0.5838	hp15	1.8200	0.1211	15.025
E113	0.1670	0.1467	1.1386	hp16	2.2274	0.0958	23.256
E114	0.0767	0.1431	0.5358	hp17	2.5339	0.0981	25.837
E115	0.4619	0.4801	0.9620	hp18	2.4830	0.0863	28.760
E116	0.0888	0.1132	0.7845	hp19	2.8817	0.1143	25.218
E117	0.1306	0.0911	1.4327	hp20	2.1726	0.1000	21.733
Hospital Dummies				hp21	2.3860	0.1051	22.707
hp1	2.4131	0.1050	22.985	hp22	2.2713	0.0954	23.814
hp2	2.7220	0.1221	22.291	hp23	2.2632	0.1156	19.576
hp3	2.4362	0.1414	17.234	hp24	2.3082	0.1205	19.158
hp4	2.3184	0.1059	21.883	hp25	2.1602	0.0783	27.574
hp5	2.8916	0.0938	30.828	hp26	2.3512	0.1915	12.279
hp6	2.5863	0.1562	16.556	hp27	2.3182	0.1814	12.780
$\widehat{\lambda}_{_{N}}$	0.3935	0.0046	86.067				
R2	0.35138						

Table 1 Results of estimation

*: Significant at the 5% level.